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Global stability of varicella model

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Abstract. Varicella is a disease caused by the varicella-zooster virus. This disease is common in children under 10 years of age and is not a fatal condition. However, some cases of varicella in adults are more dangerous because they can cause pneumonia. Here, discussion about analysis of varicella epidemic model is presented. This model is expressed in the form of 6th order differential equation with state variables as follows susceptible, exposed, infected, quarantine, recovered, and vaccination. Apart vaccination and isolation intervention, this model also consider disinfectant spray an¡d ventilation. Our analysis shows that varicella dynamic behavior depends on the basic reproduction number ($\mathcal{R}_0$). The model has two equilibria, namely, free disease and endemic equilibria. By using the Lyapunov function, we demonstrate that when $\mathcal{R}_0 \leq 1$, disease-free is globally asymptotically stable, and when $\mathcal{R}_0 > 1$ disease-free becomes unstable while endemic is globally asymptotically stable. This results indicate more effective each intervention, the better the control of varicella.

1. Introduction

Varicella (chickenpox) and zoster (shingles) are two diseases caused by the varicella-zoster virus (VZV) [1]. Varicella or chickenpox is the primary manifestation of VZV infection and characterized by mild headache, fever, malaise, aching muscle, loss of appetite, a feeling of nausea and an eruption of blisters on the skin and mucous membranes [2]. Meanwhile, zoster or shingles are caused by the reactivation of latent VZV. The reactivated virus is latent within dorsal root ganglia and goes to nerve cells and causes neural damage [3]. In general, varicella is a mild disease and this disease occurs more frequently in children. In The United Kingdom and Canada, many children under 15 years suffers by varicella, which is about 85% for both in the United Kingdom and Canada [1]. Although varicella is a minor illness in the primary infection, the superinfection that occurs could cause death, especially in adults [3]. Varicella is a highly contagious disease that is transmitted by inhalation of saliva droplets dispersed in the air by infected subjects or by direct contact with skin lesions of subjects with varicella or zoster [4]. Currently, there is no specific treatment for varicella, the main prevention is to give vaccines, especially in school-age children. Before the introduction of vaccination, varicella spread widely in Italy and The United States. In The United States, there were about 4 million cases, 11.000-13.000 cases of hospitalization and 100-150 deaths per year in early 1990s. After the first vaccines introduction, the number of cases decreased 79% in the 2000-2005, and hospital admission and death decreased 90% in the same year [4]. This means that the use of vaccines is an effective way to reduce the varicella cases. Beside vaccination, closure and isolation or spraying disinfection is needed to control the spread [5].

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In order to understand the transmission of varicella disease and to evaluate the control measure, many authors developed mathematical model such as SIR model [6], SEIR epidemic dynamic model [7], fractional MSEIR (maternally-derived immunity, susceptible, exposed, Infected, and recovered) model [2] and also modified SEIR model including the cycle of shingles acquisition [8]. Some of these models are equipped with simulations based on actual situation or curve fitting. In [7], authors didn't discuss analysis of mathematical model at all. Thus, the dynamic system behavior of this model is still not understood. This motivate us to discuss of analysis of the mathematical model [7]. To make the model more general, we extented the model by considering a recruitment factor and a natural death.

2. Model Formulation

Here, we consider the varicella model from previous work [7] where the human population is divided into six compartments, i.e., susceptible (S), exposed (E), infected (I), quarantine (Q), recovered (R) and vaccination (V). This grouping is based on their health status. Susceptible is a compartment containing healthy people. Exposed is a compartment containing infected people who are not capable infecting healthy people. This compartment indicates that the varicella virus requires an incubation period. Infectious is a compartment containing infected people who are capable infecting healthy people. Quarantine is a compartment containing isolated infected people. Quarantine or isolation are provided for some infected people to reduce the rate of disease transmission and speed up recovery of those who are sick. Recovered is a compartment containing recovering from the disease. Vaccination is a compartment containing healthy people who have been vaccinated. This model considers three interventions such as vaccination, isolation/quarantine, and finally ventilation and disinfectant spray. Each intervention is represented by the parameters δ, ψ, and m whose values are in the interval $(0, 1)$. The transmission process of varicella is illustrated by the following diagram.

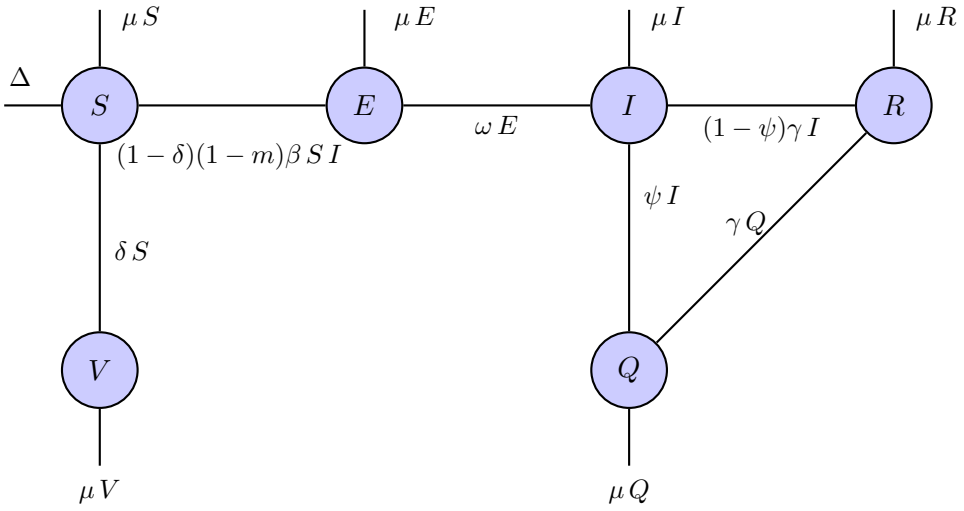

Figure 1. Varicella transmission diagram

This model is expressed in the following ordinary differential equation

$$\begin{aligned}
\dot{S} &= \Delta - (1-\delta)(1-m)\beta IS - (\mu+\delta)S \\
\dot{E} &= (1-\delta)(1-m)\beta IS - (\mu+\omega)E \\
\dot{I} &= \omega E - (\mu+\psi+(1-\psi)\gamma)I \\
\dot{Q} &= \psi I - (\mu+\gamma)Q \\
\dot{R} &= (1-\psi)\gamma I + \gamma Q - \mu R \\
\dot{V} &= \delta S - \mu V.
\end{aligned} \tag{1}$$

Suppose N is the total population. So,

$$N = S + E + I + Q + R + V. \tag{2}$$

Differentiate equation (3) respect to time give

$$\dot{N} = \dot{S} + \dot{E} + \dot{I} + \dot{Q} + \dot{R} + \dot{V} = \Delta - \mu N. \tag{3}$$

From the solution of equation (3), we have $N \to \dfrac{\Delta}{\mu}$ as $t \to \infty$.
Thus, the system (1) the domain which has biological meaning as follows

$$\Omega = \left\{ (S, E, I, Q, R, V) \in R_+^5 \mid S + E + I + Q + R + V \leq \frac{\Delta}{\mu} \right\}$$

and initial conditions

$$S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, Q(0) \geq 0, R(0) \geq 0, V(0) \geq 0.$$

Explanation of all parameters in the model are provided at Table 1.

Table 1. List of parameters of the model

Parameters	Description
β	The infection rate
ω	The latency coefficient
γ	The removal rate
m	Disinfection and ventilation efficiency
δ	The vaccination coefficient
ψ	The isolation rate
Δ	The recruitment rate
μ	The natural death rate

3. Basic Reproduction Number

The basic reproduction number, denoted by R_0, is a threshold parameter for disease growth in the population. This parameter determines whether the disease will become extinct or persist in the population. It is easy to show that the system (1) has the following zero solution

$$E_0 = \left(\frac{\Delta}{\mu + \delta}, 0, 0, 0, 0, \frac{\delta\, \Delta}{\mu(\mu + \delta)} \right).$$

E_0 is called disease free equilibrium (DFE) that represent the disease is not exist in the population. Here, we derive the formulation of basic reproduction number by following the method from [9]. Let $x = (E, I, S, Q, R, V)$, the system (1) can be rewritten in the following form

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x) \tag{4}$$

where $\mathcal{F}$ and $\mathcal{V}$ represents new infection terms and the transition terms, respectively. Hence, we obtain

$$\mathcal{F} = \begin{bmatrix} (1-\delta)S(1-m)\beta\, I \\ \omega E \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} (\mu+\omega)E \\ (\mu + \psi + (1-\psi)\gamma)I \\ -\Delta + (1-\delta)S\beta(1-m)\beta\, I + (\mu+\delta)S \\ -\psi I + (\mu+\gamma)Q \\ -(1-\psi)\gamma I - \gamma Q + \mu R \\ -\delta S + \mu V \end{bmatrix}.$$

The next step, we just consider the component $\mathcal{F}$ and $\mathcal{V}$ that contain infected and infectious. Evaluation of Jacobian matrix $\mathcal{F}$ and $\mathcal{V}$ at E_0 are given by

$$D\mathcal{F}(E_0) = \begin{bmatrix} 0 & \dfrac{\Delta\,(1-\delta)(1-m)\beta}{\mu+\delta} \\ \omega & 0 \end{bmatrix} \quad \text{and} \quad D\mathcal{V}(E_0) = \begin{bmatrix} -\mu-\omega & 0 \\ 0 & -(1-\psi)\gamma - \mu - \psi \end{bmatrix}.$$

The next generation matrix (NGM) is obtained as

$$NGM = D\mathcal{F}(E_0)\left(D\mathcal{V}(E_0)\right)^{-1} = \begin{bmatrix} 0 & \dfrac{\Delta\,(1-\delta)(1-m)\beta}{(\mu+\delta)\left((1-\psi)\gamma+\mu+\psi\right)} \\ \dfrac{\omega}{\mu+\omega} & 0 \end{bmatrix}.$$

Basic reproduction number is formulated as radius spectral of NGM. Hence, we have

$$\mathcal{R}_0 = \rho\left(NGM\right) = \sqrt{\frac{\Delta\,(1-\delta)(1-m)\beta\,\omega}{(\mu+\delta)\left((1-\psi)\gamma+\mu+\psi\right)(\mu+\omega)}}. \tag{5}$$

For convenience in the next discussion, we will use an equivalent parameter, R_0, to basic reproduction number where $R_0 = \mathcal{R}_0^2$. This imply $R_0 < 1 \iff \mathcal{R}_0 < 1$ and $R_0 > 1 \iff \mathcal{R}_0 > 1$.

4. Existence of Endemic

Let $E_1 = (S^*, E^*, I^*, R^*, Q^*, V^*)$. This is an equilibrium point of system (1) if E_1 satisfy the following equation

$$\begin{aligned} 0 &= \Delta - (1-\delta)(1-m)\beta S^* I^* - (\mu+\delta)S^* \\ 0 &= (1-\delta)(1-m)\beta S^* I^* - (\mu+\omega)E^* \\ 0 &= \omega E^* - (\mu + \psi + (1-\psi)\gamma)I^* \\ 0 &= \psi I^* - (\mu+\gamma)Q^* \\ 0 &= (1-\psi)\gamma I^* + \gamma Q^* - \mu R^* \\ 0 &= \delta S^* - \mu V^*. \end{aligned} \tag{6}$$

The first and third equation in (6) give solutions

$$S^* = \frac{\Delta}{(1-\delta)(1-m)\beta\, I^* + \delta + \mu}, \quad I^* = \frac{\omega\, E^*}{(1-\psi)\gamma + \mu + \psi}. \tag{7}$$

Substitute (7) to the second equation in (6) yields

$$E^* = \frac{\Delta\,(1-\delta)(1-m)\beta\,\omega - (\mu+\delta)\,((1-\psi)\gamma+\mu+\psi)\,(\mu+\omega)}{\omega\,\beta\,(1-\delta)(1-m)\,(\mu+\omega)}.$$

Some of substitutions give other components of E_1 as follows

$$\begin{aligned}
I^* &= \frac{\Delta\,(1-\delta)(1-m)\beta\,\omega - (\mu+\delta)\,((1-\psi)\gamma+\mu+\psi)\,(\mu+\omega)}{((1-\psi)\gamma\,\mu + (1-\psi)\gamma\,\omega + \mu^2 + \mu\,\omega + \mu\,\psi + \omega\,\psi)\,(1-\delta)(1-m)\beta} \\
S^* &= \frac{(\mu+\omega)\,((1-\psi)\gamma+\mu+\psi)}{\beta\,\omega(1-\delta)(1-m)} \\
V^* &= \frac{\delta\ (\mu+\omega)\,((1-\psi)\gamma+\mu+\psi)}{(1-\delta)(1-m)\beta\,\omega\,\mu} \\
Q^* &= \frac{\Delta\,\psi\,(1-\delta)(1-m)\beta\,\omega - \psi\ (\mu+\delta)\,((1-\psi)\gamma+\mu+\psi)\,(\mu+\omega)}{\beta\,(1-\delta)(1-m)\,(\mu+\omega)\,(\gamma+\mu)\,((1-\psi)\gamma+\mu+\psi)}.
\end{aligned} \tag{8}$$

By using the formula (5), some of the components of E_1 can be expressed in R_0 as follows

$$\begin{aligned}
E^* &= \frac{(\mu+\delta)((1-\psi)\gamma+\mu+\psi)(R_0-1)}{\beta\,\omega(1-\delta)(1-m)} \\
I^* &= \frac{(\mu+\delta)(R_0-1)}{\beta(1-\delta)(1-m)} \\
Q^* &= \frac{\psi(\mu+\delta)(R_0-1)}{\beta(1-\delta)(1-m)(\mu+\gamma)} \\
R^* &= \frac{\gamma(\mu+\delta)((1-\psi)(\mu+\gamma)+\psi)(R_0-1)}{\beta\mu(1-\delta)(1-m)(\mu+\gamma)}.
\end{aligned} \tag{9}$$

In order to have a biological meaning, then the components of E_1 in the equation (9) is required become positive. In other words, E_1 exist if $R_0 > 1$. This point represents an endemic where the disease is present for long term in the population, so that it is well knows as an endemic point.

5. Stability of Equilibria

In this section, we investigate the local stability of a disease free (E_0) and an endemic point (E_1). Evaluation of Jacobi matrix at E_0 yields

$$J(E_0) = \begin{bmatrix}
-\delta-\mu & 0 & -\dfrac{\Delta\,(1-\delta)(1-m)\beta}{\mu+\delta} & 0 & 0 & 0 \\
0 & -\mu-\omega & \dfrac{\Delta\,(1-\delta)(1-m)\beta}{\mu+\delta} & 0 & 0 & 0 \\
0 & \omega & -(1-\psi)\gamma-\mu-\psi & 0 & 0 & 0 \\
0 & 0 & \psi & -\gamma-\mu & 0 & 0 \\
0 & 0 & (1-\psi)\gamma & \gamma & -\mu & 0 \\
\delta & 0 & 0 & 0 & 0 & -\mu
\end{bmatrix}$$

with characteristic polynomial is given by

$$p(\lambda) = \frac{1}{\mu+\delta}(\lambda+\mu)^2(\lambda+\mu+\delta)(\lambda+\mu+\gamma)(a_2\lambda^2+a_1\lambda+a_0) \tag{10}$$

where

$$a_2 = (\mu+\delta)$$
$$a_1 = (\mu+\delta)(2\mu+\omega+\psi+(1-\psi)\gamma)$$
$$a_0 = -\Delta\,(1-\delta)(1-m)\beta\,\omega + (\mu+\delta)\,((1-\psi)\gamma+\mu+\psi)\,(\mu+\omega)\,.$$

The eigenvalues of Jacobi matrix, $J(E_0)$, are $-\mu, -\mu-\delta, -\mu-\gamma$ and the roots of quadratic factor. By using formula R_0, coefficient a_0 can be rewriten as follows

$$(\mu+\delta)\,(c\gamma+\mu+\psi)\,(\mu+\omega)\,(1-R_0) > 0.$$

Clearly, if $R_0 < 1$, the roots of quadratic factor are negative. Thus, we conclude if $R_0 < 1$, the point E_0 locally asymptotically stable.

Next, we investigate the stability of the endemic point. Evaluation of Jacobi matrix at E_1 give

$$J(E_1) = \begin{bmatrix} -I^*(1-\delta)(1-m)\beta-\delta-\mu & 0 & -S^*(1-\delta)(1-m)\beta & 0 & 0 & 0 \\ I^*(1-\delta)(1-m)\beta & -\mu-\omega & S^*(1-\delta)(1-m)\beta & 0 & 0 & 0 \\ 0 & \omega & -(1-\psi)\gamma-\mu-\psi & 0 & 0 & 0 \\ 0 & 0 & \psi & -\gamma-\mu & 0 & 0 \\ 0 & 0 & c\gamma & \gamma & -\mu & 0 \\ \delta & 0 & 0 & 0 & 0 & -\mu \end{bmatrix}$$

with characteristic polynomial

$$p(\lambda) = (\lambda+\mu)^2(\lambda+\mu+\gamma)(a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0) \tag{11}$$

where the coefficient a_1, a_2, a_3 and a_0 are given by

$$\begin{aligned} a_3 &= 1 \\ a_2 &= I^*(1-\delta)(1-m)\beta + c\gamma + \delta + 3\,\mu + \omega + \psi \\ a_1 &= I^*(1-\delta)(1-m)\beta\;((1-\psi)\gamma + 2\,\mu + \omega + \psi) - (1-\delta)(1-m)\beta\,\omega\,S^* \\ &\quad (\delta+\mu+\omega)\,((1-\psi)\gamma+2\,\mu+\psi) + \delta\,\omega + \mu\;((1-\psi)\gamma+\mu+\psi) \\ a_0 &= (1-\delta)(1-m)\beta\;(\mu+\omega)\,((1-\psi)\gamma+\mu+\psi)\,I^* \\ &\quad + (\mu+\delta)\,(-(1-\delta)(1-m)\beta\,\omega\,S^* + (\mu+\omega)((1-\psi)\gamma+\mu+\psi))\,. \end{aligned}$$

Equation (11) indicate eigenvalues of E_1 are $-\mu, -\mu-\gamma$ and others is the roots of cubic factor. Substitute S^* in (8) to a_0 and a_1 yields

$$a_1 = ((1-\psi)\gamma + 2\,\mu + \omega + \psi)\,(I^*(1-\delta)(1-m)\beta + \delta + \mu)$$

$$a_0 = (1-\delta)(1-m)\beta\;(\mu+\omega)\,((1-\psi)\gamma+\mu+\psi)\,I^*.$$

So, if $R_0 > 1$, then all sign of a_0, a_1 and a_2 are positive. Now, observe that

$$(1-\delta)(1-m)\beta I + \delta + \mu > (1-\delta)(1-m)\beta I, \quad 2\mu+\omega+\psi+(1-\psi)\gamma > \mu+\omega,$$

$$\text{and} \quad (1-\psi)\gamma+\psi+\delta+3\mu+\omega>(1-\psi)\gamma+\psi+\mu.$$

It follows

$$\begin{aligned}((1-\psi)\gamma+2\,\mu+\omega+\psi)\,(I^*\beta+\delta+\mu)\,(I^*(1-\delta)(1-m)\beta+c\gamma+\delta+3\,\mu+\omega+\psi)\\ >(1-\delta)(1-m)\beta\,I^*\,(\mu+\omega)\,((1-\psi)\gamma+\mu+\psi)\end{aligned}$$

or

$$a_2a_1>a_3a_0.$$

According to Routh Hurwitz criterion, that all eigenvalues are negative for cubic factor. Thus the point E_1 is locally asymptotically stable. The result is summarized in the following theorem If $R_0<1$, then E_0 is locally asymptotically stable and if $R_0>1$, then E_0 is unstable and E_1 is locally asymptotically stable.

6. Global Stability

In order to discuss the global stability of disease free equilibrium, E_0, we consider the Lyapunov function

$$V_1=\frac{\omega}{(\mu+\omega)}E+I. \tag{12}$$

Derivative of Lyapunov function respect to time a long solution of system (1) is given by

$$\begin{aligned}\dot{V}_1&=\frac{\omega}{(\mu+\omega)}\dot{E}+\dot{I}\\&=\frac{\omega}{(\mu+\omega)}\left((1-\delta)(1-m)\beta SI-(\mu+\omega)E\right)+(\omega E-(\mu+\psi+c\gamma)I)\\&=\frac{(1-\delta)(1-m)\beta SI\omega}{(\mu+\omega)}-(\mu+\psi+(1-\psi)\gamma)I\\&=\left(\frac{(1-\delta)(1-m)\beta S\omega}{(\mu+\omega)(\mu+\psi+(1-\psi)\gamma)}-1\right)(\mu+\psi+(1-\psi)\gamma)I.\end{aligned} \tag{13}$$

Due to $S\leq\dfrac{\Delta}{\mu+\delta}$, it follows

$$\dot{V}_1\leq\left(\frac{\Delta\,(1-\delta)(1-m)\beta\omega}{(\mu+\delta)(\mu+\omega)(\mu+\psi+(1-\psi)\gamma)}-1\right)(\mu+\psi+(1-\psi)\gamma)I=(R_0-1)(\mu+\psi+(1-\psi)\gamma)I$$

Clearly, if $R_0\leq 1$, then $\dot{V}\leq 0$.
Now let $X=(S,E,I,Q,R,V)$. In case $\dot{V}_1=0$, observe the largest invariant set contained in

$$\{X\in\,\Omega:\dot{V}_1=0\}\text{ is }\Omega_0=\{X\in\,\Omega:I=0)\}.$$

Since fact that $\Omega_0=E_0$ for $R_0<1$, the according to LaSalle-Lyapunov Theorem [10], as $t\,\to\infty$, then all trajectories starting in Ω toward to E_0.

In order to investigate global stability of endemic point, we consider the following Lyapunov function

$$V_2=S-S^*-S^*\ln\frac{S}{S^*}+E-E^*-E^*\ln\frac{E}{E^*}+\left(\frac{\mu+\omega}{\omega}\right)\left(I-I^*-I^*\ln\frac{I}{I^*}\right). \tag{14}$$

Derivative V_2 respect to time a long solution in system (1)

$$\begin{aligned}\dot{V}_2 =& \left(\dot{S} - \frac{S^*}{S}\dot{S}\right) + \left(\dot{E} - \frac{E^*}{E}\dot{E}\right) + \left(\frac{\mu+\omega}{\omega}\right)\left(\dot{I} - \frac{I^*}{I}\dot{I}\right) \\ =&\Delta - (1-\delta)(1-m)\beta SI - (\mu+\delta)S - \frac{S^*}{S}\left(\Delta - (1-\delta)(1-m)\beta SI - (\mu+\delta)S\right) \\ &+ (1-\delta)(1-m)\beta SI - (\mu+\omega)E - \frac{E^*}{E}\left((1-\delta)(1-m)\beta SI - (\mu+\omega)E\right) \\ &+ (\mu+\omega)E - \left(\frac{\mu+\omega}{\omega}\right)(\mu+\psi+(1-\psi)\gamma)I - \left(\frac{\mu+\omega}{\omega}\right)\frac{I^*}{I}\left(\omega E - (\mu+\psi+(1-\psi)\gamma)I\right).\end{aligned} \tag{15}$$

At equilibrium, we have

$$\Delta = (1-\delta)(1-m)\beta S^* I^* + (\mu+\delta)S^* \tag{16}$$

Substitution (16) to (15) give

$$\begin{aligned}\dot{V}_2 =& (1-\delta)(1-m)\beta S^* I^* + (\mu+\delta)S^* - (\mu+\delta)S - (1-\delta)(1-m)\beta S^* I^* \frac{S^*}{S} \\ &- (\mu+\delta)S^*\frac{S^*}{S} + (1-\delta)(1-m)\beta I S^* + (\mu+\delta)S^* - (1-\delta)(1-m)\beta SI\frac{E^*}{E} + (\mu+\omega)E^* \\ &- \left(\frac{\mu+\omega}{\omega}\right)(\mu+\psi+(1-\psi)\gamma)I - \frac{I^*}{I}(\mu+\omega)E + \left(\frac{\mu+\omega}{\omega}\right)(\mu+\psi+(1-\psi)\gamma)I^*.\end{aligned} \tag{17}$$

Since

$$(\mu+\psi+(1-\psi)\gamma) = \frac{\omega\, E^*}{I^*}, \quad (\mu+\omega) = \frac{(1-\delta)(1-m)\beta\, S^* I^*}{E^*},$$

we have

$$(1-\delta)(1-m)\beta\, S^* I - \left(\frac{\mu+\omega}{\omega}\right)(\mu+\psi+(1-\psi)\gamma)I = \left((1-\delta)(1-m)\beta\, S^* I^* - (\mu+\omega)E^*\right) I^* I = 0.$$

It follows

$$\begin{aligned}\dot{V}_2 =& (1-\delta)(1-m)\beta S^* I^* + (\mu+\delta)S^* - (\mu+\delta)S \\ &- (1-\delta)(1-m)\beta S^* I^* \frac{S^*}{S} - (\mu+\delta)S^*\frac{S^*}{S} + (\mu+\delta)S^* - (1-\delta)(1-m)\beta SI\frac{E^*}{E} \\ &+ (1-\delta)(1-m)\beta S^* I^* - \frac{I^*}{I}\frac{(1-\delta)(1-m)\beta\, S^* I^*}{E^*}E + (1-\delta)(1-m)\beta\, S^* I^*\end{aligned} \tag{18}$$

or

$$\dot{V}_2 = (1-\delta)(1-m)\beta S^* I^*\left(3 - \frac{S^*}{S} - \frac{SIE^*}{S^* I^* E} - \frac{I^* E}{IE^*}\right) + (\mu+\delta)S^*\left(2 - \frac{S}{S^*} - \frac{S^*}{S}\right). \tag{19}$$

By introducing function $g(x) = 1 - x - \ln(x)$ which is monotone decreasing for $x > 0$. Hence, $g(x) \leq 0$ for all $x > 0$. Then we can express inequality from (19) as follows

$$\left(3 - \frac{S^*}{S} - \frac{SIE^*}{S^* I^* E} - \frac{I^* E}{IE^*}\right) \leq 0 \tag{20}$$

and

$$\left(2 - \frac{S}{S^*} - \frac{S^*}{S}\right) \leq 0. \tag{21}$$

Therefore, if $R_0 > 1$, then $\dot{V} \leq 0$. Derivative Lyapunov function will be $\dot{V} = 0$ if only if $S = S^*, E = E^*, I = I^*$. Therefore, we conclude that the largest compact invariance set $\{(S, E, I) \in \Omega \mid \dot{V} = 0\}$ is the singleton E_1. Hence, LaSalle-Lyapunov Theorem [10] implies that E_1 is globally asymptotically stable in Ω. The result is summarized in the following theorem If $R_0 \leq 1$, then E_0 is globally asymptotically stable and if $R_0 > 1$, then E_0 is unstable and E_1 is globally asymptotically stable. Theorem 5 and theorem 6 indicate that the basic reproduction number (R_0) determine the dynamics behavior of varicella. Based on the theorem, in order to eliminate the disease in the population, we need to make the value of R_0 is less than 1. Since the value of R_0 is also influenced by the parameter of interventions such as δ, m and ψ, thus we can say that the intervention of vaccination, isolation and disinfectant spray and ventilation is important to control the varicella. Qualitatively, we can say the more effective each intervention, the smaller the R_0 value. This mean, the better the control of varicella. If in practice, the intervention could make $R_0 < 1$, theoretically the varicella would become extinct. On the other hand, if $R_0 > 1$, the varicella will become an endemic disease.

7. Conclusion

In this paper, we have analyzed a varicella model from previous work [7]. Our results show that basic reproduction number ($\mathcal{R}_0$) has play important role in the dynamics behavior of the the model. We found two equilibria in this model, i.e., disease free and endemic. The existence and stability of equilibria depend on $\mathcal{R}_0$. Our analysis demonstrated that disease free is globally asymptotically stable if $\mathcal{R}_0 \leq 1$ and endemic is globally asymptotically stable if $\mathcal{R}_0 > 1$. The rusults of our analysis are in accordance with previous result [8] which showed that the dynamics behavior of varicella depend on R_0. Since the value of R_0 depend on the parameter interventions such as δ, m and ψ, then the results indicate that interventions can control the varicella. To eradicate the varicella in the population, the intervention of vaccination, isolation and ventilation and disinfectant spray should effective such that the value of R_0 less than one.

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