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Output Tracking of Some Classes of Non-minimum Phase Nonlinear Systems By Redefinition Output Approach

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Abstract. In this paper, we present the redefinition output approach for output tracking of some classes of non-minimum phase nonlinear systems without and with uncertainty. Input-Output linearization and gradient descent methods are applied to design the control. The good result of this approach is demonstrated by 3 examples.

1. Introduction

A system is called non-minimum phase if a nonlinear state feedback can hold the system output identically zero while the internal dynamics becomes unstable [1]. Output tracking problem for nonlinear non-minimum phase systems is a rather difficult issue in control theory. Most of researcher restrict their research to a special class nonlinear system only. The input-output linearization is one of the most available methods [1] for minimum phase besides the modified gradient descent control [2]. In this paper, both methods are applied to design control after non-minimum phase nonlinear system is transformed to minimum phase by redefinition output of the system. Results on stabilization of non-minimum phase system in the output feedback form have been presented in [3], [4], [5]. The main idea in [3], [4], [5] is output reconstruction such that the original nonlinear systems becomes minimum phase with respect to a new output. Results on output tracking of some classes of non-minimum phase nonlinear system have been presented in [6], [7]. In [6], The design of the input control is based on the exact linearization.

In this paper we give 3 examples to demonstrate how to track the output of the system by redefinition output combine with input-output linearization or modified gradient descent control. First example, the nonlinear system is assumed exact linearizable, the second example, the nonlinear system has the relative degree is $n - 1$, n is the dimension of the system. The third example, the nonlinear system has the relative degree is $n - 1$ with uncertainty parameter. For relative degree is 1, Dimitar Ho and J.Karl Hedrick has done in [8].



2. Problem Statement

Consider the following SISO affine nonlinear control system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \theta) + \mathbf{g}(\mathbf{x}, \theta)u, \tag{1}$$

$$y = h(\mathbf{x}) \tag{2}$$

where $\mathbf{x} \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}$ is the control input, $y \in \mathcal{R}$ is the measured output, and θ is the parameter uncertainty. $\mathbf{f} : \mathcal{R}^n \rightarrow \mathcal{R}^n$ is a smooth function with $\mathbf{f}(0) = 0$, $\mathbf{g} : \mathcal{R}^n \rightarrow \mathcal{R}^n$ and $h : \mathcal{R}^n \rightarrow \mathcal{R}$ are smooth functions. Assume also that $h(0) = 0$. If the nonlinear system (1)-(2) has relative degree r , ($r < n$) at x^o , by input-output linearization [1], the system (1)-(2) can be transformed to

$$S = \begin{cases} \sum_{ext} : \begin{cases} \dot{\xi}_k = \xi_{k+1}, & k = 1, \dots, r-1 \\ \dot{\xi}_r = a(\xi, \eta, \theta) + b(\xi, \eta, \theta)u \end{cases} \\ \sum_{int} : \dot{\eta} = q(\xi, \eta, \theta) \\ y = \xi_1 = h(\mathbf{x}), \end{cases} \tag{3}$$

with the internal dynamics

$$\sum_{int} : \dot{\eta} = q(\xi, \eta, \theta). \tag{4}$$

The stability of the internal state η is required to guarantee the output system $y(t)$ tracks the desired output $y_d(t)$. Our objective is to design input such that the output $y(t)$ tracks the desired output $y_d(t)$ while keeping the state bounded.

For case $b(\xi, \eta, \theta) \neq 0$ for $t \geq 0$, we apply input-output linearization method, i.e.,

$$u_r = \frac{1}{b(\mathbf{z})} (-a(\mathbf{z}) + v), \tag{5}$$

where $\mathbf{z} = [\xi_1, \dots, \xi_r, \eta_1, \dots, \eta_{n-r}]$, $v = c_0z_1 + c_1\dot{z}_2 + \dots + c_nz_1^{(n)}$ and the value of $c_i; i = 0, \dots, n$ is chosen such that the real part of the eigen values of polynomial $p(s)$

$$p(s) = c_n s^n + c_{n-1} s^{n-1} + \dots + c_1 s + c_0$$

are negative, $z_1 = h(\mathbf{x})$.

For case $b(\xi, \eta, \theta) = 0$ for a time t , we apply the modified gradient descent control[7]. Let $\eta(t)$ is a virtual output of the systems and $\eta_d(t)$ is the virtual desired output, and equilibrium point of the internal dynamics of normal form of the system. Then we find r_η as relative degree of the system if $\eta(t)$ is the output of the system. We know that $\eta \in \mathcal{R}^{n-r}$ then $r_\eta = [r_\eta^1, \dots, r_\eta^{n-r}]$.

Based on $y(t)$, $\eta(t)$ and their derivatives, we construct the performance index as a descent function as follows,

$$F_0(y(t), \eta(t)) = \left(\sum_{j=0}^r a_j (y^{(j)}(t) - y_d^{(j)}(t)) \right)^2 + \sum_{i=1}^{n-r} \left(\sum_{j=0}^{r_\eta^i} b_j^i (\eta_i^{(j)}(t) - \eta_{di}^{(j)}(t)) \right)^2, \tag{6}$$

where the constants $a_0, \dots, a_r; b_0^i, \dots, b_{r_\eta^i}^i, i = 1, \dots, n-r$ will be chosen such that eigenvalues of polynomials

$$a_r s^r + a_{r-1} s^{r-1} + \dots + a_1 s + a_0, \tag{7}$$

$$b_{r_\eta^i}^i s^{r_\eta^i} + b_{r_\eta^i-1}^i s^{r_\eta^i-1} + \dots + b_1^i s + b_0^i, \tag{8}$$

are real negative. Defines the descent function F_0 as a quadratic function to ensure that the function F_0 has a minimum value.

The modified gradient descent control is

$$\dot{u} = -\frac{dF_0}{du} + v, \quad (9)$$

where v is an artificial input,

$$v = \begin{cases} k(\mathbf{x}, u) & \text{if } \frac{\partial F_0}{\partial u} \neq 0 \\ 0 & \text{if } \frac{\partial F_0}{\partial u} = 0, \end{cases} \quad (10)$$

with

$$k(\mathbf{x}, u) = \frac{1}{\frac{\partial F_0}{\partial u}} \left(-\frac{\partial F_0}{\partial x} \dot{x} - \sqrt{\left(\frac{\partial F_0}{\partial x} \dot{x} \right)^2 + \left(\frac{\partial F_0}{\partial u} \right)^2} \right). \quad (11)$$

3. Redefinition Output Approach

3.1. Exact Linearization

Consider the following SISO affine nonlinear control system

$$\begin{aligned} \dot{x}_1 &= x_2 + 2x_1^2 \\ \dot{x}_2 &= x_3 + u \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{x}_3 &= x_1 + x_3 \\ y &= x_1; \quad y_d(t) = \sin t. \end{aligned} \quad (13)$$

By using the output $\lambda(x) = x_3$, the nonlinear system (12) can be linearized exactly.

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= a(\mathbf{z}) + u, \end{aligned} \quad (14)$$

where $a(\mathbf{z}) = z_1 + z_2 + (2(z_2 - z_1) + 1)(z_3 - z_2 - 2(z_2 - z_1)^2 + 2(z_2 - z_1)^2)$. By input-output linearization technique we get

$$u = -a(\mathbf{z}) + v. \quad (15)$$

Let $y_d(t) = \sin(t) = x_{1d}(t)$. Next, we choose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$ then $y(t)$ tracks the desired output $y_d(t)$. Consider the equation : $\dot{x}_3 = x_1 + x_3$. By replacing x_1 with $x_{1d}(t) = \sin(t)$, we have a differential equation $\dot{x}_3 - x_3 = \sin(t)$. Then, we solve the differential equation to obtain $x_3 = 1/2(-\sin(t) - \cos(t))$. This solution we state as $x_{3d}(t) = 1/2(-\sin(t) - \cos(t))$. Thus, for the output tracking problem we have

$$v = \frac{1}{a_3} \dot{z}_{3d} - \sum_{i=1}^3 a_{i-1}(z_i - z_{id}). \quad (16)$$

The simulation results is given in Figure 1.

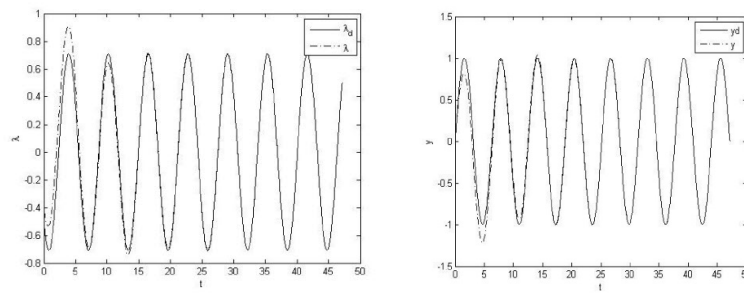


Figure 1. Left : Output Tracking (exact linearization), x_3 to x_{3d} ; Right: Output Tracking (original system) y to y_d

3.2. Relative degree $r = n - 1$

Consider the nonlinear system (SISO)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 + x_1x_3 \\ \dot{x}_3 &= x_4 - u + x_1x_3 \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{x}_4 &= u - 2x_1x_3 \\ y &= x_1. \end{aligned} \tag{18}$$

The nonlinear system (17)-(18) has relative degree 3 at any point x_0 (relative degree of the system is not well defined). Because the stability of zero dynamic is unstable, the nonlinear system (17)-(18) is the non-minimum phase. Now, redefining output $z_1 = x_1 + 2x_2 + 2x_3 + 2x_4$.

By considering the new output, the relative of the system (17) is 3 at any point x_0 ((relative degree of the system is not well defined). The system (17) in normal form with respect to output z_1

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= a(z) + b(z)u \\ \dot{\eta} &= -\eta + z_3, \end{aligned} \tag{19}$$

with $a(z) = x_4 - x_1^2x_3 - 3x_1x_3 - x_2x_3 - x_1x_4$, $b(z) = 1 + x_1$. Thus the system (17) is the minimum phase with respect to the new output.

Then according to (9), the modified steepest descents control with respect to z_1 is

$$\dot{u} = -2(1 + x_1)a_3 \left(a_0(z_1 - z_{1d}) + a_1(\dot{z}_1 - \dot{z}_{1d}) + a_2(\ddot{z}_1 - \ddot{z}_{1d}) + a_3 \left(z_1^{(3)} - z_{1d}^{(3)} \right) \right) + v, \tag{20}$$

where v as in equation (10).

Let $y_d(t) = x_{1d}(t) = 0.5\sin(t)$. Next, we choose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$, then $y(t)$ tracks the desired output $y_d(t)$. By replacing x_1 with $x_{1d}(t) = 0.5\sin(t)$, then $x_{2d} = 0.5\cos(t)$. By replacing x_2 with $x_{2d}(t)$, then $x_{3d} = -\frac{0.5\sin(t)}{1+0.5\sin(t)}$. By replacing x_3 with $x_{3d}(t)$, we have a differential equation $\dot{x}_4 - x_4 = \frac{-0.5\cos(t)}{(1+0.5\sin(t))^2} + \frac{0.25\sin^2(t)}{1+0.5\sin(t)}$. Thus $x_{4d} = 1/2(-0.5\cos(t) - 0.5\sin(t) + \frac{\sin(t)}{1+0.5\sin(t)})$. Now, $z_{1d} = 0.5\cos(t)$.

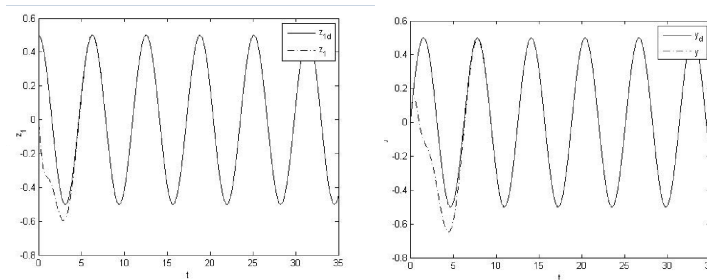


Figure 2. Left : Output Tracking, z_1 to z_{1d} ; Right: Output Tracking (original system) y to y_d

Simulation results for the modified steepest descent control (20) are shown in Figure 2 for constants $a_0 = 15$, $a_1 = 23$, $a_2 = 9$, $a_3 = 1$. Initial value $x_1(0) = 0$, $x_2(0) = 0.5$, $x_3(0) = 0$, $x_4(0) = -0.5$, $u(0) = 0.2$:

3.3. Relative degree $r = n - 1$ with uncertainty

Consider the following SISO affine nonlinear control system

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1^3 \\ \dot{x}_2 &= x_3 - u + 2x_1^3 \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{x}_3 &= \theta \sin(x_1) + u - 2x_1^3 \\ y &= x_1. \end{aligned} \tag{22}$$

The zero dynamic system (21)-(22) is $\eta = \dot{\eta}$. Thus the system (21)-(22) is the non-minimum phase.

Now redefining output : $z_1 = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$. Then we have the zero dynamic system (21)-(22) with respect to the output z_1 is

$$\dot{\eta} = \eta - \left(\frac{-\eta}{\alpha - 1} \right) - \left(\frac{-\eta}{\alpha - 1} \right)^3 + \theta \sin \left(\frac{-\eta}{\alpha - 1} \right),$$

and

$$\begin{aligned} \eta \dot{\eta} &= \eta^2 + \frac{\eta^2}{\alpha - 1} + \frac{\eta^4}{(\alpha - 1)^3} + \eta \theta \sin \left(\frac{-\eta}{\alpha - 1} \right) \\ &\leq \eta^2 + \frac{\eta^2}{\alpha - 1} + \frac{\eta^4}{(\alpha - 1)^3} + |\eta| |\theta| \left| \frac{-\eta}{\alpha - 1} \right| \\ &= \frac{\eta^2 (|\theta| - \alpha)}{|\alpha - 1|} + \frac{\eta^4}{\alpha - 1}. \end{aligned} \tag{23}$$

If $|\theta| \leq \alpha$, then $\eta \dot{\eta} < 0$. Thus the system (21) with respect to the output z_1 is minimum phase. Let $y_d(t) = \pi/2$. By replacing x_1 with $x_{1d} = y_d = \pi/2$, then $x_{2d} = (\pi/2)^3$. By replacing x_2 with x_{2d} , we have a differential equation $\dot{x}_3 - x_3 = \theta$. Thus $x_{3d} = -\theta$. Now, $z_{1d} = \alpha x_{1d} + x_{2d} + x_{3d} = \alpha(\pi/2) + (\pi/2)^3 - \theta$.

The modified steepest descent control with respect to the output z_1 is

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z}_1 - \dot{z}_{1d}) + a_2(\ddot{z}_1 - \ddot{z}_{1d}))(1 - \alpha) + v, \tag{24}$$

where v as in equation (10).

Simulation results are shown in Figure 3 for constants $a_0 = 12$, $a_1 = 14$, $a_2 = 6$, $\alpha = 0.75$. Initial value $x_1(0) = 0$, $x_2(0) = 1$, $x_3(0) = 0$, $u(0) = 0$, $\theta(t) = 0.6$.

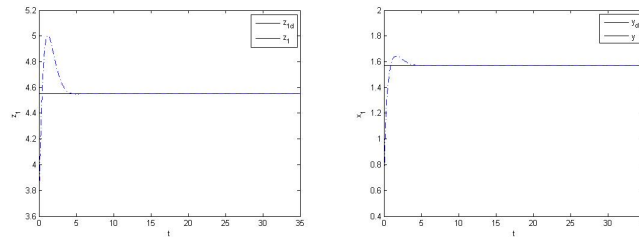


Figure 3. Left : Output Tracking, x_3 to x_{3d} ; Right: Output Tracking (original system) y to y_d

4. Conclusion

In this paper, we have applied the input-output linearization and modified gradient descent control for 3 examples of the nonlinear nonminimum phase systems with or without uncertainty. Before applying both of method, the output of the system must be redefined as a linear combination of the state variables systems, such that the system becomes minimum phase with respect to a new output. Furthermore, the new desired output will be set based on the desired output of the original system. Simulation results are shown that the output of the systems tracks the output desired of the systems. From these results, it is possible to apply both methods to any relative degree of the nonlinear control system.

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