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Design of Vaccination and Fumigation on Host-Vector Model by Input-Output Linearization Method

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Abstract. Here, we analyze the Host-Vector Model and proposed design of vaccination and fumigation to control infectious population by using feedback control especially input-output liniearization method. Host population is divided into three compartments: susceptible, infectious and recovery. Whereas the vector population is divided into two compartment such as susceptible and infectious. In this system, vaccination and fumigation treat as input factors and infectious population as output result. The objective of design is to stabilize of the output asymptotically tend to zero. We also present the examples to illustrate the design model.

INTRODUCTION

Diseases transmitted by vector known as vector born disease. Aedes mosquito is the most common vector that responsible to Chikungunya, Dengue, Rift Valley Fever, Yellow Fever and Zika. Others such as Culex mosquito responsible to Japanese Encephalitis, Lymphatic Filariasis dan West Nile Fever. Next, a familiar and dangerous disease is Malaria that caused by Anopheles mosquito [31]. Class of the diseases are health problems that threatens the entire of the world. Each year, incidence of the diseases up to 1 billion cases and 1 million of them died as well as currently, half human population living in areas where dengue is endemic [31], so they are at risk of the disease. The spread of the disease has been studied through mathematical modeling. It is useful tool to describe the dynamics of host and vector population. In addition it can give benefit to design of the diseases control. In the literature, we can found many studies of the spread of the diseases [2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 20, 23, 27, 29, 30]. Models that studied the spread of the diseases in the presence of the control can be found in [1, 2, 17, 18, 21, 24, 25]. In these studies, control approaching using constant parameter or optimal control theory.

In this work, we propose design of vaccination and fumigation on Host-Vector Model by approaching feedback control especially input-output linearization method and consider human as host and mosquito as a vector. Organization of this paper are as follows. The second section describe a model SIR-SI. The third section discuss analysis model including positivity and boundedness as well as local stability. In the next section, we discuss design of vaccination and fumigation. In the fifth section, we present numerical results to support control design from previous section. In the last section, we discuss the whole of the paper and further research.

MODEL

Here, in the Host-Vector Model, population of host is denoted by x_h and population of vector is denoted by x_v. Host population is divided by three compartments such as susceptible, infectious and recovery individuals that denoted by x_1, x_2 and x_3 respectively. Whereas the vector population is divided by two compartment such as susceptible and infectious individuals that denoted by x_4 and x_5, respectively. Therefore, $x_h = x_1 + x_2 + x_3$ and $x_v = x_4 + x_5$. The model is expressed in the form of system differential equation

$$\dot{x}_1 = a_1 - b_1 x_1 x_5 - (c_1 + u_1) x_1 \tag{1}$$

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$$\dot{x}_2 = b_1 x_1 x_5 - (c_1 + d_1)\, x_2 \quad (2)$$
$$\dot{x}_3 = d_1 x_2 - c_1 x_3 + u_1 x_1 \quad (3)$$
$$\dot{x}_4 = a_2 - b_2 x_2 x_4 - (c_2 + u_2)\, x_4 \quad (4)$$
$$\dot{x}_5 = b_2 x_2 x_4 - (c_2 + u_2)\, x_5 \quad (5)$$

with initial condition $x_1(0), x_2(0), x_3(0), x_4(0), x_5(0) \geq 0$, and the domain is $\Omega = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5_+ \mid x_1 + x_2 + x_3 = x_h, x_4 + x_5 = x_v\}$. The description of all parameters are provided in the following table.

TABLE 1. Parameters Description

Parameters	Description	unit
a_1	host recruitment rate	day^{-1}
b_1	contact rate between infectious vector and susceptible host	day^{-1}
c_1	natural death rate of host	day^{-1}
d_1	transmission rate from infectious to recovery	day^{-1}
a_2	vector recruitment rate	day^{-1}
b_2	contact rate between infectious host and susceptible vector	day^{-1}
c_2	natural death rate of vector	day^{-1}
u_1	fraction of vaccination	-
u_2	fraction of fumigation	-

For case the vector is mosquito, contact rate is formulated by $b_1 = \frac{bp_h x_1 x_5}{x_h}$, and $b_2 = \frac{bp_v x_2 x_4}{x_h}$ where b is biting rate, p_h and p_v are transmission of probability.

POSITIVITY AND BOUNDEDNESS

The positivity and boundedness condition of model (1)-(5) described by the following theorems.

Theorem 1 *Let* $x_1(0), x_2(0), x_3(0), x_4(0), x_5(0) \geq 0$*, then solution of* $(1) - (5)$ *in* Ω *for* $t > 0$

Proof. Let $\mathbf{u}_i$ is vector normal to axis of x_i where $i = 1, 2, 3, 4, 5$ with direct to interior Ω. Take $\mathbf{u}_i$ as follows $\mathbf{u}_1 = (0, 1, 1, 1, 1), \mathbf{u}_2 = (1, 0, 1, 1, 1), \mathbf{u}_3 = (1, 1, 0, 1, 1), \mathbf{u}_4 = (1, 1, 1, 0, 1), \mathbf{u}_5 = (1, 1, 1, 1, 0)$.
Next, calculate dot product between u_i and $\mathbf{x}$
$\mathbf{u}_1 \cdot \dot{\mathbf{x}} = 0{\cdot}\dot{x}_1 + \dot{x}_2 + \dot{x}_3 + \dot{x}_4 + \dot{x}_5 = a_2 + u_1 x_1 > 0$
$\mathbf{u}_2 \cdot \dot{\mathbf{x}} = \dot{x}_1 + 0{\cdot}\dot{x}_2 + \dot{x}_3 + \dot{x}_4 + \dot{x}_5 = a_1 + a_2 > 0$
$\mathbf{u}_3 \cdot \dot{\mathbf{x}} = \dot{x}_1 + \dot{x}_2 + 0{\cdot}\dot{x}_3 + \dot{x}_4 + \dot{x}_5 = a_1 + a_2 > 0$
$\mathbf{u}_4 \cdot \dot{\mathbf{x}} = \dot{x}_1 + \dot{x}_2 + \dot{x}_3 + 0{\cdot}\dot{x}_4 + \dot{x}_5 = a_1 > 0$
$\mathbf{u}_5 \cdot \dot{\mathbf{x}} = \dot{x}_1 + \dot{x}_2 + \dot{x}_3 + \dot{x}_4 + 0{\cdot}\dot{x}_5 = a_1 + a_2 > 0$
The values of dot product is positive, then direct of vector fields is to interior of Ω,
$\therefore$ solution of (1)-(5) is positive for $t > 0$. This prove the theorem

Theorem 2 *There is* $M > 0$ *such that the solution* $(1) - (5)$ *satisfies* $x_i < M, i = 1, 2, 3, 4, 5$

Proof. Let $\chi_1(t) = x_1(t) + x_2(t) + x_3(t)$. Differentiate χ_1 to t, we have

$$\begin{aligned}
\dot{\chi}_1 &= \dot{x}_1 + \dot{x}_2 + \dot{x}_3 \\
&= a_1 - b_1 x_1 x_5 - (c_1 + u_1)\, x_1 + b_1 x_1 x_5 - (c_1 + d_1)\, x_2 + d_1 x_2 - c_1 x_3 + u_1 x_1 \\
&= a_1 - c_1 (x_1 + x_2 + x_3) \\
&= a_1 - c_1 \chi_1 \\
\dot{\chi}_1 &= a_1 - c_1 \chi_1 \implies 0 < \chi_1(t) = \frac{a_1}{c_1} + \chi_1(0)\, e^{-c_1 t} \quad \text{where} \quad \chi_1(0) = x_1(0) + x_2(0) + x_3(0)
\end{aligned}$$

Next for t tend to infinity, we have

$$limsup_{t \to \infty} \chi_1(t) = \frac{a_1}{c_1}, \quad \implies \quad 0 < x_1 + x_2, x_3 \leq \frac{a_1}{c_1}$$

Let $\chi_2 = x_4 + x_5$. Differentiate χ_2 to t, Then we have

$$\begin{aligned}
\dot{\chi}_2 &= \dot{x}_4 + \dot{x}_5 \\
&= a_2 - b_2 x_2 x_4 - (c_2 + u_2) x_4 + b_2 x_2 x_4 - (c_2 + u_2) x_5 \\
&= a_2 - (c_2 + u_2)(x_4 + x_5) \\
\dot{\chi}_2 &= a_2 - (c_2 + u_2)\chi_2 \quad \Longrightarrow \quad 0 < \chi_2(t) = \frac{a_2}{c_2 + u_2} + \chi_2(0)\, e^{-(c_2+u_2)t} \quad \text{where} \quad \chi_2(0) = x_4(0) + x_5(0)
\end{aligned}$$

Next for t tend to infinity, we have

$$limsup_{t\to\infty}\chi_2(t) = \frac{a_2}{c_2 + u_2}, \quad \Longrightarrow \quad 0 < x_4(t), x_5(t) \leq \frac{a_2}{c_2 + u2}$$

Take $M = \max\{\frac{a_1}{c_1}, \frac{a_2}{c_2+u_2}\}$, then $x_i(t) < M$ where $i = 1, 2, 3, 4, 5$. This prove the theorem.

EQUILIBRIUM POINTS AND IT'S STABILITY

To find equilibrium points, set $\dot{x}_i(t) = 0$, where $i = 1, 2, 3, 4, 5$. By using eliminations, we have the following quadratic equation

$$b_2(c_1 + d_1)(a_2 b_1 + c_1 c_2 + c_2 u_1) x_2{}^2 + (-a_1 a_2 b_1 b_2 + c_2 (c_1 + u_1)(c_1 + d_1)(c_2 + u_2)) x_2 = 0$$

The solutions are

$$x_2 = 0, \text{ and } x_2 = \frac{a_1 a_2 b_1 b_2 - c_2 (c_1 + u_1)(c_1 + d_1)(c_2 + u_2)}{b_2 (c_1 + d_1)(a_2 b_1 + c_1 c_2 + c_2 u_1)}$$

using algebraic manipulation, we have complete solutions as follows

$$E_0 = \left\{\frac{a_1}{\eta}, 0, \frac{u_1 a_1}{\eta c_1}, \frac{a_2}{\tau}, 0\right\} \quad \text{dan} \quad E_1 = \left\{x_1^*, x_2^*, x_3^*, x_4^*, x_5^*\right\}$$

where

$$\begin{aligned}
x_1^* &= \frac{\tau^2\sigma + a_1\tau b_2}{\eta\tau b_2 + a_2 b_2 b_1}, \quad x_2^* = \frac{\eta\tau^2(R_0 - 1)}{\eta\tau b_2 + a_2 b_2 b_1}, \quad x_3^* = \frac{\tau((R_0 - 1)\eta\tau d_1 + \sigma\tau u_1 + u_1 a_1 b_2)}{b_2(\eta\tau + a_2 b_1) c_1} \\
x_4^* &= \frac{\eta\tau\sigma + a_2\sigma b_1}{\sigma\tau b_1 + a_1 b_1 b_2}, \quad x_5^* = \frac{\eta\tau\sigma\,(R_0 - 1)}{\sigma\tau b_1 + a_1 b_1 b_2}
\end{aligned}$$

and

$$R_0 = \frac{a_1 a_2 b_1 b_2}{(c_1 + u_1)(c_2 + u_2)^2 (d_1 + u_1)}, \quad \eta = c_1 + u_1, \quad \tau = c_2 + u_2, \quad \sigma = c_1 + d_1$$

E_0 is free disease point where there are no infectious population either host or vector. E_0 is endemic point where it will exist if $R_0 > 1$. R_0 is a threshold parameter that determines existence of E_1 as well as stability of E_0 and E_1. Stability of E_0 is given by the following theorem

Theorem 3 *Equilibrium E_0 is locally stable in Ω if $R_0 < 1$ and unstable saddle if $R_0 > 1$*
Proof. Evaluation of Jacobian Matrix at E_0 gives

$$J_0 = \begin{bmatrix}
-c_1 - u_1 & 0 & 0 & 0 & -\frac{b_1 a_1}{c_1 + u_1} \\
0 & -d_1 - c_1 & 0 & 0 & \frac{b_1 a_1}{c_1 + u_1} \\
u_1 & d_1 & -c_1 & 0 & 0 \\
0 & -\frac{b_2 a_2}{c_2 + u_2} & 0 & -c_2 - u_2 & 0 \\
0 & \frac{b_2 a_2}{c_2 + u_2} & 0 & 0 & -c_2 - u_2
\end{bmatrix} \tag{6}$$

Characteristic polynomial of J_0 is

$$p_0 = (\lambda + c_1)(\lambda + c_1 + u_1)(\lambda + c_2 + u_2)\left(\lambda^2 + (c_1 + c_2 + d_1 + u_2)\lambda + (c_2 + u_2)(d_1 + c_1)(1 - R_0)\right)$$

The eigenvalues of p_0 are $-c_1, -c_1 - u_1, -c_2 - u_2$ and root of the quadratic equation. Since values of all parameters is positif, then according to Hurwitz Criteria if $R_0 < 1$, then the part of real root is negative. Therefore, the point of E_0 is locally stable if $R_0 < 1$ and saddle point if $R_0 > 1$. This proven the theorem 3.

Theorem 4 *Equilibrium E_1 is locally stable in Ω if $R_0 > 1$*
Proof. Evaluation of Jacobian Matrix at E_1 gives

$$J_1 = \begin{bmatrix} -\frac{b_1\eta\tau\sigma M}{\sigma\tau b_1 + a_1 b_1 b_2} - c_1 - u_1 & 0 & 0 & 0 & -\frac{b_1(\sigma\tau^2 + \tau a_1 b_2)}{\eta\tau b_2 + a_2 b_1 b_2} \\ \frac{b_1\eta\tau\sigma M}{\sigma\tau b_1 + a_1 b_1 b_2} & -d_1 - c_1 & 0 & 0 & \frac{b_1(\sigma\tau^2 + \tau a_1 b_2)}{\eta\tau b_2 + a_2 b_1 b_2} \\ u_1 & d_1 & -c_1 & 0 & 0 \\ 0 & -\frac{b_2(\eta\sigma\tau + \sigma a_2 b_1)}{\sigma\tau b_1 + a_1 b_1 b_2} & 0 & -\frac{b_2\eta\tau^2 M}{\eta\tau b_2 + a_2 b_1 b_2} - c_2 - u_2 & 0 \\ 0 & \frac{b_2(\eta\sigma\tau + \sigma a_2 b_1)}{\sigma\tau b_1 + a_1 b_1 b_2} & 0 & \frac{b_2\eta\tau^2 M}{\eta\tau b_2 + a_2 b_1 b_2} & -c_2 - u_2 \end{bmatrix} \tag{7}$$

Characteristic polynomial of J_1 is

$$p_1 = (\lambda + c_1)(\lambda + c_2 + u_2)\left(\lambda^3 + s_1\lambda^2 + s_2\lambda + s_3\right)$$

where

$$\begin{aligned} s_1 &= \frac{\eta\tau\left(\eta\sigma\tau + \sigma\tau^2 + \sigma a_2 b_1 + \tau a_1 b_2\right)(R_0 - 1)}{(\sigma\tau + a_1 b_2)(\eta\tau + a_2 b_1)} + 2c_1 + c_2 + d_1 + u_1 + u_2 \\ s_2 &= \frac{\eta\tau\left(\sigma a_2(\tau + \sigma) b_1 + \tau a_1(\eta + \sigma) b_2 + \sigma\tau(\eta\sigma + \eta\tau + \sigma\tau + 2\tau c_1)\right)(R_0 - 1)}{(\sigma\tau + a_1 b_2)(\eta\tau + a_2 b_1)} \\ &\quad + \frac{\eta^2\sigma\tau^3(R_0 - 1)^2}{(\sigma\tau + a_1 b_2)(\eta\tau + a_2 b_1)} + (c_1 + u_1)(c_1 + c_2 + d_1 + u_2) \\ s_3 &= \frac{\eta^2\sigma\tau^3(d_1 + c_1)(R_0 - 1)^2}{(\sigma\tau + a_1 b_2)(\eta\tau + a_2 b_1)} + \frac{\eta\tau(d_1 + c_1)\left(2\tau^2\sigma\eta + (\eta a_1 b_2 + \sigma a_2 b_1)\tau\right)(R_0 - 1)}{(\sigma\tau + a_1 b_2)(\eta\tau + a_2 b_1)} \end{aligned}$$

The eigenvalues are $-c_1, -c_2 - u_2$ and root of third order polynomial with coefficient s_1, s_2, s_3. All parameters are positive, if $R_0 > 1$, then $a_1, a_2, a_3 > 0$. After some arrangement, we have

$$s_1 s_2 - s_3 > \frac{1}{((\tau\sigma + a_1 b_2)(\eta\tau + a_2 b_1))^2}\left(\eta^3\sigma\tau^4\left(\eta\sigma\tau + \sigma\tau^2 + \sigma a_2 b_1 + \tau a_1 b_2\right)(R_0 - 1)^3\right) > 0$$

According to Hurwitz Criteria, the polynomial has part of real root is negative. Therefore, E_1 is locally stable if $R_0 > 1$. This proven the theorem.

CONTROL DESIGN

The method in this section refers in [14]. System (1)-(5) can be simplified in the form

$$\dot{x} = f(x) + g(x)u, \quad y = h(x)$$

where

$$f(x) = \begin{bmatrix} a_1 - b_1 x_1 x_5 - c_1 x_1 \\ b_1 x_1 x_5 - (c_1 + d_1) x_2 \\ d_1 x_2 - c_1 x_3 \\ a_2 - b_2 x_2 x_4 - c_2 x_4 \\ b_2 x_2 x_4 - c_2 x_5 \end{bmatrix}, \quad g(x) = \begin{bmatrix} -x_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & 0 & 0 \\ 0 & 0 & 0 & -x_4 & 0 \\ 0 & 0 & 0 & 0 & -x_5 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ 0 \\ u_1 \\ u_2 \\ u_2 \end{bmatrix}$$

y is output, since our goal is to control the infectious population, then we choose $y = x_2$. Differentiate the output, yield

$$\dot{y} = L_f h(x) + L_g h(x) u$$

if $L_g h(x) = 0$, differentiate again until rth derivative where $L_g L_f^{r-1} h(x) \neq 0$, we have

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x) u \quad \text{with} \quad L_g L_f h(x) = \frac{\partial (L_f) h}{\partial x} g(x) \quad \text{and} \quad L_f^r h(x) = \frac{\partial (L_f^{r-1} h)}{\partial x} f(x)$$

The control law is $u = \frac{1}{L_g L_f^{r-1} h(x)} \left(-L_f^r h(x) + v \right)$, r is called relative degree.
For design vaccination, set $u_2 = 0$. Next, differentiate the output until linear relation between output and input reached

$$\begin{aligned}
\dot{y} &= b_1 x_1 x_5 - (c_1 + d_1) x_2 \\
\ddot{y} &= b_1 \dot{x}_1 x_5 + b_1 x_1 \dot{x}_5 - (c_1 + d_1) \dot{x}_2 \\
\dddot{y} &= b_1 x_5 (a_1 - b_1 x_1 x_5 - c_1 x_1 - u_1 x_1) + b_1 x_1 (b_2 x_2 x_4 - c_2 x_5) - (c_1 + d_1)(b_1 x_1 x_5 - (c_1 + d_1) x_2)
\end{aligned}$$

The control law is

$$u_1 = \frac{v_1 + f_1}{b_1 x_1 x_5} \tag{8}$$

where $f_1 = (d_1 + c_1)^2 x_2 + (b_2 x_2 x_4 - x_5 (2 c_1 + c_2 + d_1)) b_1 x_1 + b_1 x_5 (a_1 - b_1 x_1 x_5)$
State transformation $T(x) = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & h(x) & L_f h(x) \end{bmatrix}^T$, where ϕ_1, ϕ_2, ϕ_3 is smooth functions that satisfy $\frac{\partial \phi}{\partial x} g = 0$ and $T(x)$ is invertible

$$\begin{aligned}
\frac{\partial \phi_1}{\partial x} g &= 0 \iff -\frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_1}{\partial x_3} = 0 \implies \phi_1 = x_4 \\
\frac{\partial \phi_2}{\partial x} g &= 0 \iff -\frac{\partial \phi_2}{\partial x_1} + \frac{\partial \phi_2}{\partial x_3} = 0 \implies \phi_2 = x_5 \\
\frac{\partial \phi_3}{\partial x} g &= 0 \iff -\frac{\partial \phi_3}{\partial x_1} + \frac{\partial \phi_3}{\partial x_3} = 0 \implies \phi_3 = x_1 + x_3
\end{aligned}$$

Then, the state transformation is

$$z = T(x) = \begin{bmatrix} x_4 \\ x_5 \\ x_1 + x_3 \\ x_2 \\ b_1 x_1 x_5 - d_1 x_2 \end{bmatrix} = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \xi_1 \\ \xi_2 \end{bmatrix} \tag{9}$$

From (3), we have normal form as follows

$$\dot{\eta}_1 = a_2 - b_2 \eta_1 \xi_1 - c_2 \eta_1,\ \dot{\eta}_2 = b_2 \eta_1 \xi_1 - c_2 \eta_2,\ \dot{\eta}_3 = a_1 - \xi_2 - c_1 \eta_3,\ \dot{\xi}_1 = \xi_2,\ \dot{\xi}_2 = v_1,\ y = h(x) = \xi_1 \tag{10}$$

Here, we can choose $v_1 = -k_1 \xi_1 - k_2 \xi_2$, where k_1 and k_2 are positive parameters.
For design fumigation, set $u_1 = 0$. As previous steps, we have control law as below

$$u_2 = \frac{v_2 + f_2}{b_1 x_1 x_5} \tag{11}$$

where

$$f_2 = (d_1 + c_1)^2 x_2 + (b_2 x_2 x_4 - x_5 (2 c_1 + c_2 + d_1 + 1)) b_1 x_1 + b_1 x_5 (a_1 - b_1 x_1 x_5)$$

and the following normal form

$$\dot{\eta}_1 = a_1 - \xi_2 - d_1 \xi_1 - c_1 \eta_1,\ \dot{\eta}_2 = d_1 \xi_1 - c_1 \eta_2,\ \dot{\eta}_3 = b_2 \xi_1 + b_2 \xi_1 \eta_3 - \frac{a_2 b_1 \eta_1 \eta_3^2}{d_1 \xi_1 + \xi_2},\ \dot{\xi}_1 = \xi_2 - c_1 \xi_1,\ \dot{\xi}_2 = v_2 \tag{12}$$

we can choose $v_2 = -k_3 \xi_1 - k_4 \xi_2$, where k_3 and k_4 are positive parameters.

EXAMPLES

In this section, we consider Dengue Fever as example. Dengue Fever is vector borne disease where the virus transmitted by bitten of mosquito, particularly Aedes aegypti. In this section, we will apply the input-output linearization method to control the epidemic of the disease. Data $b = 1, p_h = 0.5, p_v = 0.33, N_h = 2000, N_v = 1000, \mu_v = 1/30, mu_h = 1/(365{\times}60), \alpha = 1/15$; Simulation of dynamics of host population in the presence of vaccination is shown in Figure 1 (a). We show a comparison the dynamics either without or with vaccination in Figure 1 (b). This figure shows clearly that vaccination is able to control the infectious population. Figure 1(c) indicate the simulation of vaccination fraction versus time. Here, we also present the basic reproduction number R_0 in time and All the time, the value is less than 1. In Figure 1 (e) and Figure 1 (f), we shows that positive parameter of k_1 and k_2 has significant effect to the infectious dynamics. For the fumigation case, the simulation is shown in Figure (2).

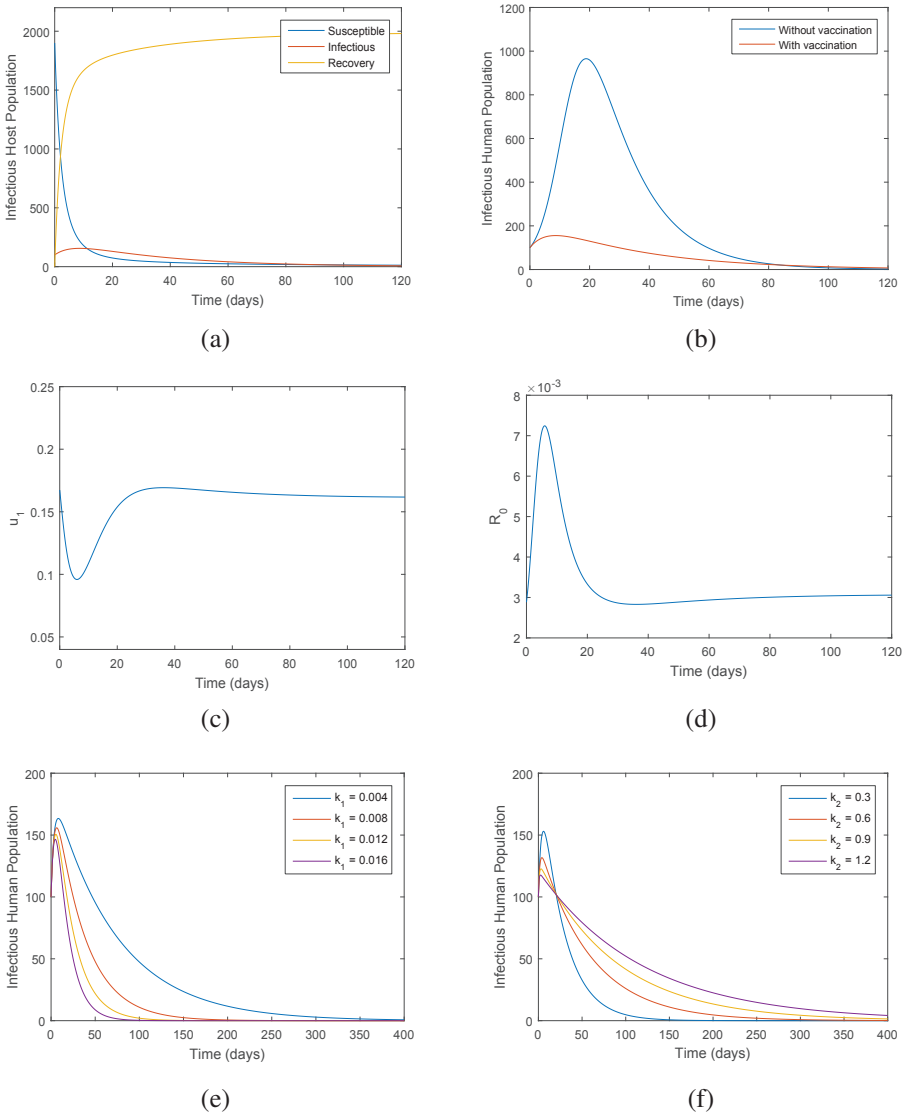

FIGURE 1. Simulation of the effect of vaccination. (a) Dynamics of human population , (b) Dynamics of infectious human population, (c) Vaccination control (u_1) versus t, (d) Reproduction number (R_0) versus t, (e) Parameter of k_1 varying, (f) Parameter of k_2 varying

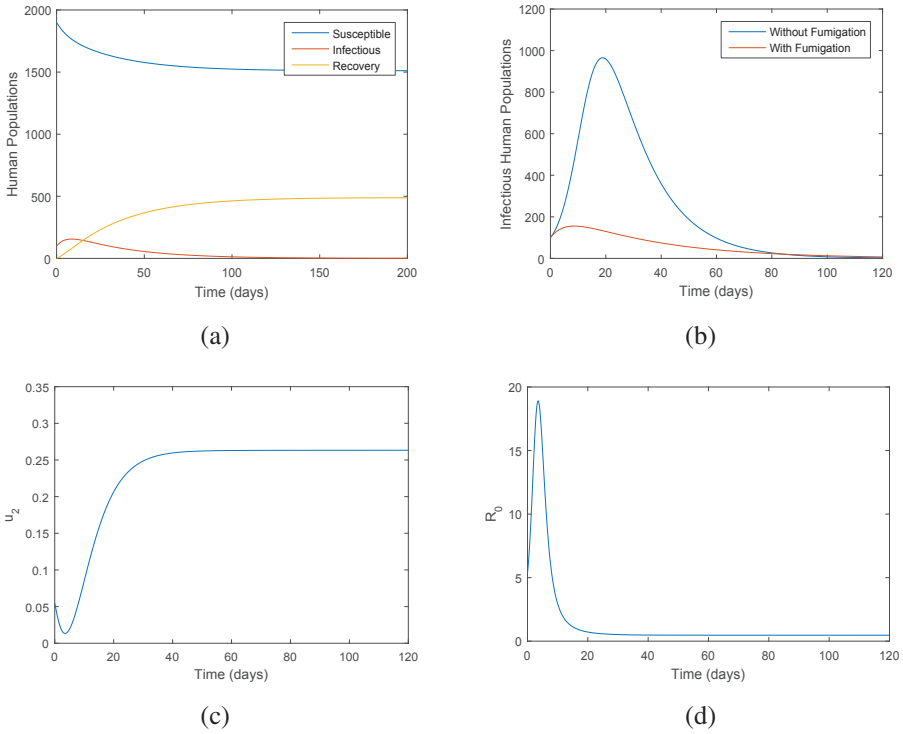

FIGURE 2. Fumigation Effect. (a) Dynamics of Human Population, (b) Dynamics of Infectious Human Population, (c) Fumigation control (u_1) versus t, (d) Reproduction number (R_0) versus t

DISCUSSION AND CONCLUSION

We have analyzed Host-Vector Model and proposed design of vaccination and fumigation by using input-output linearization method. The results show the design successful to control of epidemic the disease, see Figure 1(b) and Figure 2 (b). Vaccination can reduce the Basic Reproduction Number, with vaccination, $R_0 < 1$ all the time, see Figure 1 (d), while without control, $R_0 = 37.125$. This mean that the epidemic of the disease will be eliminated and the endemic of the disease is not exist. In the design of control law, the parameter of k_1 and k_2 is important to control the infectious population as we can see in Figure 1 (e) and Figure 1 (f). In addition, we obtain that after day 25 in vaccination, about 15 % susceptible need get vaccine to control the infection population. Whereas, in fumigation, about 25 % need to be killed to control one. The result in Figure 2 (d) shows that, in the presence of fumigation, the values R_0 is increasing until to 18.9, but decreasing significantly and after day 25, the value become 0.47. Due to less than 1, it indicate that the disease will be eliminated.

In the future, we will extend this work by including the cost of the control either vaccination or fumigation. We also interest to find formulation of u_1 and u_2 as function of t and then apply them to origin model.

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